Centre Number			Candidate Number		
Surname					
Other Names					
Candidate Signature					

A	Q	A	
		_	

General Certificate of Education Advanced Level Examination June 2014

Mathematics

MPC3

Unit Pure Core 3

Tuesday 10 June 2014 9.00 am to 10.30 am

For this paper you must have:

• the blue AQA booklet of formulae and statistical tables. You may use a graphics calculator.

Time allowed

• 1 hour 30 minutes

Instructions

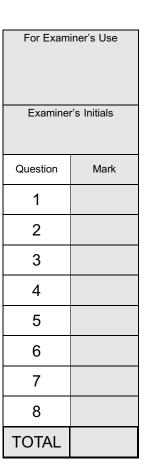
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.



Answer all questions.

Answer each question in the space provided for that question.

1 Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^\pi x^{\frac{1}{2}} \sin x \, dx$$

Give your answer to four significant figures.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 1



QUESTION PART REFERENCE	Answer space for question 1



2	A curve I	has equation	$v = 2 \ln(1)$	2e - x).

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) Find an equation of the normal to the curve $y = 2 \ln(2e - x)$ at the point on the curve where x = e.

[4 marks]

- (c) The curve $y = 2 \ln(2e x)$ intersects the line y = x at a single point, where $x = \alpha$.
 - (i) Show that α lies between 1 and 3.

[2 marks]

(ii) Use the recurrence relation

$$x_{n+1} = 2\ln(2e - x_n)$$

with $x_1=1$ to find the values of x_2 and x_3 , giving your answers to three decimal places.

[2 marks]

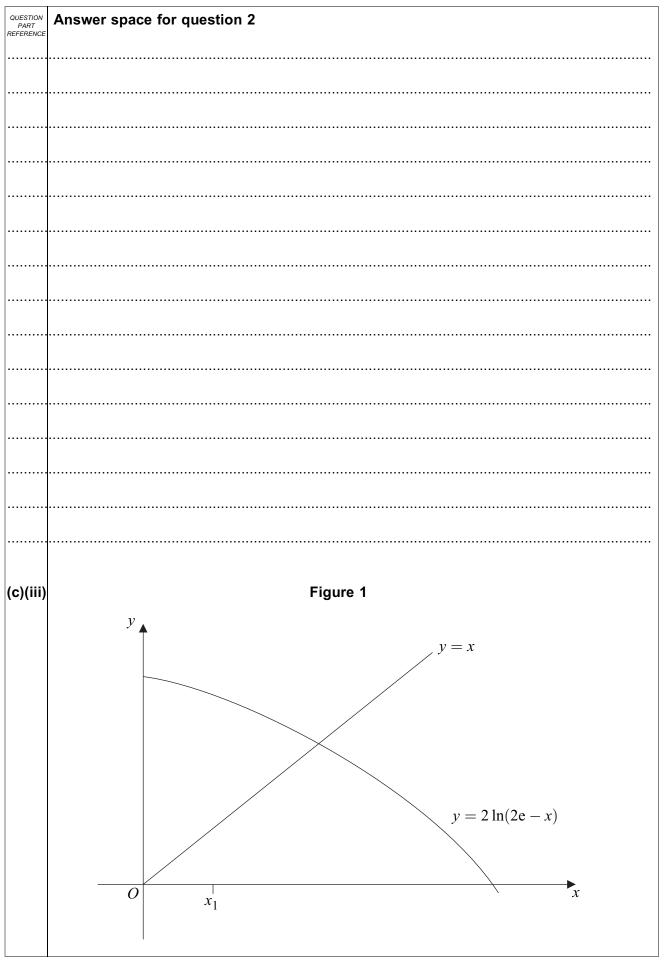
(iii) Figure 1, on the opposite page, shows a sketch of parts of the graphs of $y = 2 \ln(2e - x)$ and y = x, and the position of x_1 .

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 2







3 (a) (i) Differentiate $(x^2 + 1)^{\frac{5}{2}}$ with respect to x.

[2 marks]

(ii) Given that $y = e^{2x}(x^2 + 1)^{\frac{5}{2}}$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when x = 0.

[3 marks]

(b) A curve has equation $y = \frac{4x - 3}{x^2 + 1}$. Use the quotient rule to find the *x*-coordinates of the stationary points of the curve.

[5 marks]

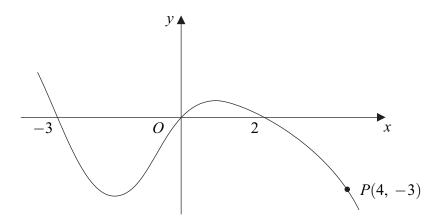
QUESTION PART REFERENCE	Answer space for question 3



QUESTION PART REFERENCE	Answer space for question 3



4 The sketch shows part of the curve with equation y = f(x).



(a) On Figure 2 below, sketch the curve with equation y = -|f(x)|.

[3 marks]

- (b) On Figure 3 on the page opposite, sketch the curve with equation y=f(|2x|). [2 marks]
- (c) (i) Describe a sequence of two geometrical transformations that maps the graph of y=f(x) onto the graph of y=f(2x+2).

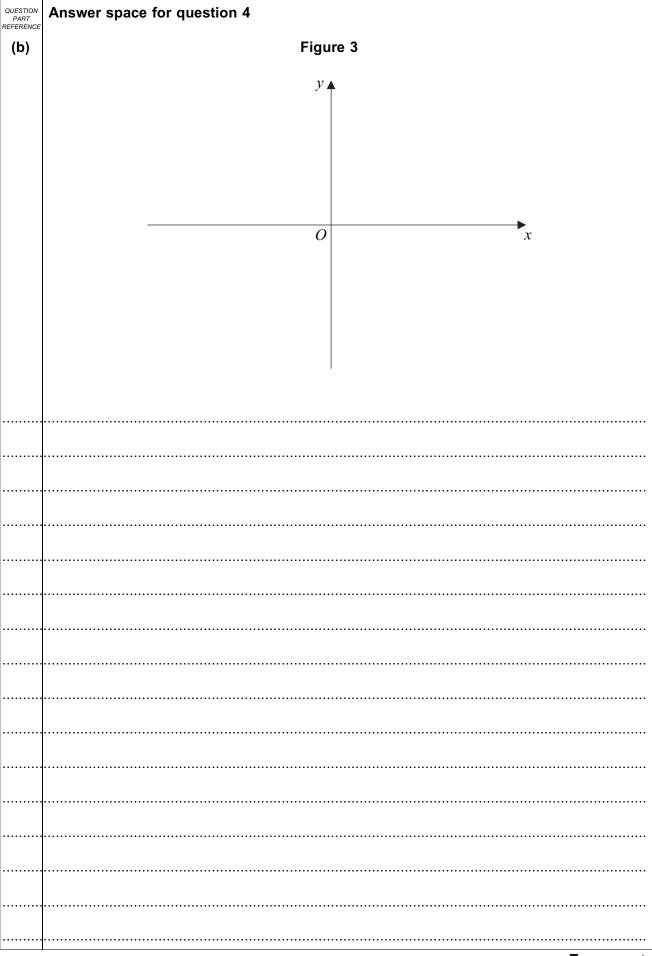
[4 marks]

(ii) Find the coordinates of the image of the point P(4, -3) under the sequence of transformations given in part (c)(i).

[2 marks]



9





5	The functions f and g are de	fined with their respective domains by
---	------------------------------	--

$$f(x) = x^2 - 6x + 5$$
, for $x \ge 3$
$$g(x) = |x - 6|$$
, for all real values of x

(a) Find the range of f.

[2 marks]

(b) The inverse of f is f^{-1} .

Find $f^{-1}(x)$. Give your answer in its simplest form.

[4 marks]

(c) (i) Find gf(x).

[1 mark]

(ii) Solve the equation gf(x) = 6.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5



6 (a) By using integration by parts twice, find

$$\int x^2 \sin 2x \, dx$$

[6 marks]

(b) A curve has equation $y = x\sqrt{\sin 2x}$, for $0 \le x \le \frac{\pi}{2}$.

The region bounded by the curve and the x-axis is rotated through 2π radians about the x-axis to generate a solid.

Find the exact value of the volume of the solid generated.

[3 marks]

QUESTION PART REFERENCE	Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6



7	Use the substitution $u = 3 - x^3$ to find the exact value of	$\int_0^1 \frac{x^5}{3 - x^3} \mathrm{d}x.$ [6 mag)	arks]
QUESTION PART	Answer space for question 7		
REFERENCE			



QUESTION PART REFERENCE	Answer space for question 7



8 (a)	Show that the expression	$1 - \sin x$	$\cos x$	can be writte	writton	26	2 sec x
o (a)	Show that the expression	$\cos x$	$1 - \sin x$	Call De	WIILLEII	as	Z SCC X

[4 marks]

(b) Hence solve the equation

$$\frac{1-\sin x}{\cos x} + \frac{\cos x}{1-\sin x} = \tan^2 x - 2$$

giving the values of x to the nearest degree in the interval $0^{\circ} \leqslant x < 360^{\circ}$.

[6 marks]

(c) Hence solve the equation

$$\frac{1 - \sin(2\theta - 30^{\circ})}{\cos(2\theta - 30^{\circ})} + \frac{\cos(2\theta - 30^{\circ})}{1 - \sin(2\theta - 30^{\circ})} = \tan^{2}(2\theta - 30^{\circ}) - 2$$

giving the values of θ to the nearest degree in the interval $0^{\circ} \leqslant \theta \leqslant 180^{\circ}$.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 8



QUESTION PART REFERENCE	Answer space for question 8



QUESTION PART REFERENCE	Answer space for question 8



QUESTION PART REFERENCE	Answer space for question 8
	END OF QUESTIONS
	FIAD OL MOFGLIONO





